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Not One Consul Kumaraswamy Dist.

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Abstract: In this research, the distribution of Consul Kumaraswamy [1] was reconstructed again, and thus a simple error was discovered in the formula for the resulting distribution, so all researchers who used this distribution in their research and studies should reconsider again.

Keywords: Consul Kumaraswamy, Reconstructed

INTRODUCTION

In recent years, we note the great interest of researchers in obtaining a new probability distribution by using many techniques. The complex distribution is produced when all or some of the original distribution parameter differ.

One of these complex distributions is the distribution of consul kumaraswamy, which was published by researcher Adil Rashid and Jan.

Consul Kumaraswamy Dist. (CKSD)

Consul distribution introduced by Consul and Shenton was modified by Islam and Consul (1990) who derived it as a bunching model in traffic flow through the branching process and also discussed its applications to automobile insurance claims and vehicle bunch size data [1].

The probability density function of the consul distribution:

$$f_1 = \begin{cases} \frac{1}{x}\binom{mx}{x-1}p^{x-1}(q)^{mx-x+1} \\ 0 \qquad otherwise \end{cases} \quad \dots 1$$

$$0 < p < 1$$
$$1 \le m \le p^{-1}$$
$$x = 1,2,\dots$$

Kumaraswamy Dist. (KSD)

A continuous probability distribution consisting of two parameters, which was proposed by the scientist Bundi kumaraswamy, one of the great engineering scientists in India [2], is similar to the beta distribution, but the reason for its opposite is that it contains a closed form of the cumulative distribution function, which makes it easier to deal with it [1].

Consul Kumaraswamy Dist. (CKSD)

If the random variable Xi follows the consul distribution with parameter m and b, where the parameter m represents a constant value, and the parameter p represents a random variable that follows the kumaraswamy distribution, so to determine the distribution resulting from marginalization on p as a composite of the consul distribution with the kumaraswamy distribution on it, the probability function of the composite CD is (m,p) with KSD(α,β) as follows:

$$f_{CKSD}(x;m,\propto,\beta) = \binom{mx}{x-1}\sum_{j=0}^{mx-x+1}\binom{mx-x+1}{j}(-1)^j B\left(\beta,\frac{x+j+\alpha-1}{\propto}+1\right) \quad \dots 2$$

$$x = 1,2,\dots.m \qquad m,\propto,\beta > 0$$

Proof: With the help of definition of proposed distribution the probability function of a compound of CD (m, p) with KSD (α,β) can be obtained as

$$f_{CKSD}(x;m,\beta,\alpha) = \int_0^1 f_1(x|p)f_2(p)dp$$

$$f_{CKSD}(x;m,\beta,\alpha) = \frac{\beta\alpha}{x}\binom{mx}{x-1}\int_0^1 p^{x+\alpha-2}(1-p)^{mx-x+1}(1-p^\alpha)^{\beta-1}dp$$

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$$f_{CKSD}(x;m,\beta,\alpha)=\frac{\beta\alpha}{x}\binom{mx}{x-1}\sum_{j=0}^{\infty}\binom{mx-x+1}{j}(-1)^j\int_0^1 p^{x+j+\alpha-2}\,(1-p^\alpha)^{\beta-1}dp$$

Substituting , $1-p^\alpha=z$ we get

$$f_{CKSD}(x;m,\beta,\alpha)=\frac{\beta\alpha}{x}\binom{mx}{x-1}\sum_{j=0}^{\infty}\binom{mx-x+1}{j}(-1)^j\int_0^1 z^{\beta-1}\,(1-z)^{\frac{x+j+\alpha-1}{\alpha}}dz$$

$$f_{CKSD}(x;m,\beta,\alpha)=\frac{\beta\alpha}{x}\binom{mx}{x-1}\sum_{j=0}^{\infty}\binom{mx-x+1}{j}(-1)^j\,B(\beta,\frac{x+j+\alpha-1}{\alpha}+1)$$

If $m\in N$ the above probability function takes the simpler rearranged form as

$$f_{CKSD}(x;m,\beta,\alpha)=\frac{\beta\alpha}{x}\binom{mx}{x-1}\sum_{j=0}^{mx-x+1}\binom{mx-x+1}{j}(-1)^j\,B(\beta,\frac{x+j+\alpha-1}{\alpha}+1)$$

Correction of the distribution of Consul Komaraswamy

The complex distribution function is equation No. (2), we have re-combined it again and some simple errors were discovered:

$$f(X;m,\alpha,\beta)=\int_0^1 f_1(x|p)f_2(p)dp$$

$$f(X;m,\alpha,\beta)=\frac{\alpha\beta}{x}\binom{mx}{x-1}\int_0^1 p^{x+\alpha-2}(1-p)^{mx-x+1}(1-p^\alpha)^{\beta-1}\,dp$$

We know that

$$(1-p)^{mx-x+1}=\sum_{j=0}^{mx-x+1}\binom{mx-x+1}{j}(-1)^j p^j$$

When substituting, we get

$$f(X;m,\alpha,\beta)=\frac{\alpha\beta}{x}\binom{mx}{x-1}\int_0^1\sum_{j=0}^{mx-x+1}\binom{mx-x+1}{j}(-1)^j\,p^{x+\alpha+j-2}(1-p^\alpha)^{\beta-1}$$

let's say

$$1-p^\alpha=z\ \rightarrow 1-z=p^\alpha\rightarrow(1-z)^{\frac{1}{\alpha}}=p$$

$$dp=\frac{1}{\alpha}(1-z)^{\frac{1}{\alpha}-1}\,dz$$

$$f(X;m,\alpha,\beta)=\frac{\alpha\beta}{x}\binom{mx}{x-1}\int_0^1\sum_{j=0}^{mx-x+1}\binom{mx-x+1}{j}(-1)^j\,(1-z)^{\frac{x+\alpha+j-2}{\alpha}}\,Z^{\beta-1}\ \frac{1}{\alpha}(1-z)^{\frac{1}{\alpha}-1}\,dz$$

$$f(X;m,\alpha,\beta)=\frac{\alpha\beta}{x}\binom{mx}{x-1}\int_0^1\sum_{j=0}^{mx-x+1}\binom{mx-x+1}{j}(-1)^j\,(1-z)^{\frac{x+\alpha+j-1}{\alpha}}\,Z^{\beta-1}\ \frac{1}{\alpha}\,dz$$

When arranging the equation, we get

$$f(X;m,\alpha,\beta)=\frac{\beta}{x}\binom{mx}{x-1}\sum_{j=0}^{mx-x+1}\binom{mx-x+1}{j}(-1)^j\int_0^1(1-z)^{\frac{x+\alpha+j-1}{\alpha}}\,Z^{\beta-1}\ \ dz$$

The final formula for Consul Komaraswamy Distribution (CKSD) is as follows:

$$f(X;m,\alpha,\beta)=\frac{\beta}{x}\binom{mx}{x-1}\sum_{j=0}^{mx-x+1}\binom{mx-x+1}{j}(-1)^j\ \ B\left(\beta,\frac{x+\alpha+j-1}{\alpha}\right)\qquad\ldots\ \ 3$$

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